

# Effects of Rotary Inertia on the Supersonic Flutter of Sandwich Panels

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A theoretical analysis is presented for the supersonic flutter of a simply-supported isotropic sandwich panel. The aerodynamic loading is described by a static approximation valid for a wide range of Mach number. Small deflection theory for sandwich plates which considers transverse shear deformations and rotary inertia in addition to bending is used. The purpose of this study is to demonstrate the effect of the rotary inertia parameter on the critical flutter speed of simply-supported panels. Previous investigators have neglected this effect and results obtained are of an anomalous nature for certain plate parameters. The effect of such plate parameters as rotary inertia, transverse shear, and midplane stress resultants on the critical flutter speed are presented in graphical form.

## Nomenclature

$A, B, C$	= constants used in solution for $w, Q_x, Q_y$ , respectively
$\bar{A}$	= $[\theta^2/(1 - rk_x)] [k_x - 2 + J\Omega^2 + r(\Omega^2 + \bar{N}_y + k_x)]$
$a$	= panel length
$\bar{B}$	= $[\theta^4/(1 - rk_x)] [r(\Omega^2 + \bar{N}_y) + \Omega^2 + \bar{N}_y - 1 + J\Omega^2]$
$b$	= panel width
$D$	= bending stiffness of isotropic sandwich
$D_Q$	= shear stiffness of isotropic sandwich
$D_1, D_2, D_3$	= coefficients of cubic equation
$d$	= frequency parameter = $(Jb^2/D\pi^2)\omega^2$
$h$	= thickness of core
$I_c$	= moment of inertia of core
$I_s$	= moment of inertia of cover sheets
$k_x$	= midplane stress coefficient = $N_y b^2/\pi^2 D$
$J$	= rotary inertia constant = $\rho_s I_s + \rho_c I_c$
$\bar{J}$	= rotary inertia parameter = $(Jb^2/D\pi^2)\omega_r^2$
$L$	= aerodynamic loading
$M$	= Mach number
$M_x, M_y$	= bending moments/unit length
$M_{xy}$	= twisting moment/unit length
$m$	= roots of determinantal equation
$N_x, N_y$	= in-plane normal loadings
$N_{xy}$	= in-plane shear loading
$\bar{N}_y$	= midplane stress parameter = $N_y b^2/\pi^2 D$
$Q_x, Q_y$	= transverse shear forces
$q$	= dynamic pressure of airstream
$r$	= shear flexibility parameter = $\pi^2 D/b^2 D_Q$
$t_s$	= thickness of both cover sheets
$u, v, w$	= displacements in $x, y, z$ coordinate directions, respectively
$x, y, z$	= coordinates
$\alpha, \sigma, \epsilon$	= roots of characteristic equation
$\beta$	= $(M^2 - 1)^{1/2}$
$\beta_i$	= $m_i/\pi\theta$
$\theta$	= $a/b$ , aspect ratio of the plate
$\lambda$	= dynamic pressure parameter = $2qb^3/\beta D$
$\lambda_{cr}$	= critical dynamic pressure parameter
$\bar{\lambda}$	= $[\lambda\theta^3(1 + r)/(1 - rk_x)]$
$\mu$	= Poisson's ratio
$\gamma$	= $[\lambda r\theta/4\pi^2(1 - rk_x)]$
$\gamma$ subs	= shearing strains with respect to subscripts
$\rho_c$	= mass density of core material
$\rho_s$	= mass density of cover sheets
$\rho_m$	= $\rho_s t_s + \rho_c h$

$t$	= time
$\phi$	= frequency parameter = $(\omega/\omega_r)^2 + (N_y b^2/\pi^2 D)$
$\omega$	= circular frequency
$\Omega^2$	= frequency parameter = $(\omega/\omega_r)^2$
$\omega_r^2$	= circular frequency of infinitely long plate, $y = \infty$
$\alpha_0, \beta_0$	= values of shear angles; associated with $\gamma_{xz}, \gamma_{yz}$
$\psi_i$	= $[r\pi^2\theta^2 d/(\pi^2\theta^2 - m_i^2 - \pi^2\theta^2 d)]$
,	= commas denote differentiation with respect to the subscripts that follow

## Introduction

EVER since aircraft and space vehicles first exceeded the speed of sound, panel flutter has been an important structural problem. This phenomenon is a self-excited oscillation of the external surface of the vehicle, resulting from dynamic interaction of aerodynamic, inertial, and elastic forces of the structural system.

This type of instability was first encountered in flight by the German V-2 missiles. After several structural failures, experimental investigations were launched to verify the existence of panel flutter and to provide some information concerning the effects of aspect ratio, thickness, and differential pressure. At the same time, panel flutter became the object of many theoretical investigations, but these inquiries were restricted basically to semi-infinite panels or panels on many supports. The application of a two-dimensional static approximation for the flutter analysis of a homogeneous isotropic plate was accomplished in 1956 by Hedgepeth.<sup>1,2</sup> Hedgepeth's application of a two-dimensional static approximation for the aerodynamic loading greatly simplified the flutter analysis since closed form solutions could be obtained for finite length panels. However, it was believed that such an analysis was only valid for a small range of Mach number and panel geometry; therefore, more detailed solutions of the panel flutter problem were initiated by including three dimensional unsteady aerodynamics<sup>3</sup> instead of the static approximation. Since the expressions obtained by using three-dimensional unsteady aerodynamics are so much more complicated, it was difficult to attain a wide region of convergence. Bohon and Dixon<sup>4</sup> have shown that the use of the more exacting aerodynamics is not necessary provided windspeeds of Mach 1.3 or greater were developed for panels of aspect ratios  $(a/b) \geq 1$ .

McElman<sup>5</sup> analyzed the flutter problem for a sandwich plate using a two-mode Galerkin solution. In 1965, Erickson and Anderson<sup>6</sup> obtained a closed form solution to the flutter problem of a sandwich plate simply-supported on its streamwise edges and either simply-supported or fixed along its leading and trailing edges. Anderson obtained solutions

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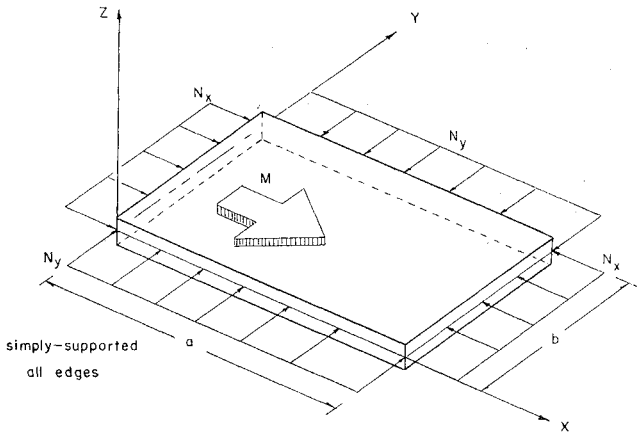


Fig. 1 Sandwich panel with stream direction, simply-supported edges, and loading.

for a wide variety of plate geometry and stress conditions. However somewhat disturbing results were obtained for panels with tensile loads applied in a direction parallel to airflow. In these cases it is theoretically possible for a reduction in transverse shear stiffness to make a panel less susceptible to flutter. The purpose of this analytic study is to investigate the effect of rotary inertia on the critical dynamic pressure parameter for these in-plane loaded sandwich plates.

### Technical Approach

#### Mathematical Model

A rectangular isotropic sandwich panel simply-supported on all edges is exposed on one surface to a supersonic airstream with flow direction parallel to the  $x$ -coordinate axis (Fig. 1). The panel may be loaded by uniform in-plane direct forces which are assumed to remain constant during deformation. In-plane shearing forces are taken equal to zero. The solution consists of evaluating the airstream speed, at which, for a panel with given geometry and externally applied stress conditions, divergent motion or flutter occurs. The following assumptions help to make the solution more amenable. The small deflection theory for sandwich panels presented by Libove and Batdorf<sup>7</sup> will be used. The equations developed are reducible to those for solid plates if the shear stiffness  $D_Q$  is considered infinite. In classical plate

theory, it is assumed that plane sections remain plane after bending. In sandwich plate theory, plane sections are assumed to remain plane but not perpendicular to the deformed middle surface. Since straight line elements do not remain perpendicular, the angles made with the deformed middle surface represent the transverse shearing strain and are denoted by  $\gamma_{xz} = \alpha_0$  and  $\gamma_{yz} = \beta_0$  in the strain displacement relations. These shearing strains are represented by a shear force divided by the shear stiffness  $D_Q$  and are a measure of the average shear angle. The aerodynamic load is given by a two-dimensional static approximation of Ackeret Theory<sup>8</sup> as follows

$$L = (2q/\beta)w_x \quad (1)$$

where  $q$  is the dynamic pressure of the stream,  $\beta^2 = M^2 - 1$  where  $M$  is the Mach Number, and  $w$  is the lateral deflection. From Eq. (1), the loading at each instant is taken as the loading which would result from flow over a stationary surface with the same shape as that of the deflected surface at that instant. Thus, no time-dependent effects are considered in the aerodynamic loading.

#### Equilibrium Equations and Boundary Conditions

The equilibrium equations and natural boundary conditions are obtained by using Hamilton's Principle. If the equations obtained are modified by taking  $N_{xy} = 0$ ,  $N_x = N_y = \text{constant}$ , and neglecting in-plane inertia and the effect of shear on rotary inertia, the equilibrium equations and natural boundary conditions become

$$\frac{Q_x}{D} = \frac{1}{D_Q} \left[ Q_{x,xx} + \frac{1-\mu}{2} Q_{x,yy} + \frac{1+\mu}{2} Q_{y,xy} \right] - w_{,xxx} - w_{,xyy} + \frac{\rho_s I_s + \rho_c I_c}{D} w_{,xtt} \quad (2)$$

$$\frac{Q_y}{D_Q} = \frac{1}{D_Q} \left[ Q_{y,yy} + \frac{1-\mu}{2} Q_{y,xx} + \frac{1+\mu}{2} Q_{x,xy} \right] - w_{,yyy} - w_{,yxx} + \frac{\rho_s I_s + \rho_c I_c}{D} w_{,yxt} \quad (3)$$

$$-Q_{x,x} - Q_{y,y} - N_x w_{,xx} - N_y w_{,yy} + (\rho_s t_s + \rho_c h) w_{,tt} + (2q/\beta)w_x = 0 \quad (4)$$

where  $N_x$  and  $N_y$  are positive in compression, and

$$\begin{aligned} Q_x &= G_c h \alpha_0 & N_x &= (Et_s/1 - \mu^2)(\epsilon_x + \mu \epsilon_y) \\ Q_y &= G_c h \beta_0 & N_y &= (Et_s/1 - \mu^2)(\epsilon_y + \mu \epsilon_x) \\ I_c &= \int_{\text{core}} z^2 dz & N_{xy} &= Gt_s(v_x + u_y) \end{aligned} \quad (5)$$

The natural boundary conditions for the problem are

$$\begin{aligned} Q_x + N_x w_{,x} + N_{xy} w_{,y} &= 0 & \text{or } \delta w &= 0 \\ M_x &= 0 & \text{or } \delta[w_{,x} - (Q_x/D_Q)] &= 0 \\ M_{xy} &= 0 & \text{or } \delta[w_{,y} - (Q_y/D_Q)] &= 0 \\ N_x &= 0 & \text{or } u &= 0 \\ N_{xy} &= 0 & \text{or } v &= 0 \end{aligned} \quad (6)$$

where

$$\begin{aligned} M_x &= -D \left[ \left( w_{,xx} - \frac{Q_{x,x}}{D_Q} \right) + \mu \left( w_{,yy} - \frac{Q_{y,y}}{D_Q} \right) \right] \\ M_y &= -D \left[ \left( w_{,yy} - \frac{Q_{y,y}}{D_Q} \right) + \mu \left( w_{,xx} - \frac{Q_{x,x}}{D_Q} \right) \right] \\ M_{xy} &= \frac{D(1-\mu)}{2} \left[ 2w_{,xy} - \frac{Q_{y,x}}{D_Q} - \frac{Q_{x,y}}{D_Q} \right] \end{aligned} \quad (7)$$

It should be noted that Eqs. (7) reduce to the equations for a solid plate if  $D_Q$  becomes infinite.

Since in practice simple-support edges involve supporting the panel throughout its thickness and not just along its midplane, for the leading and trailing edges (parallel to  $y$

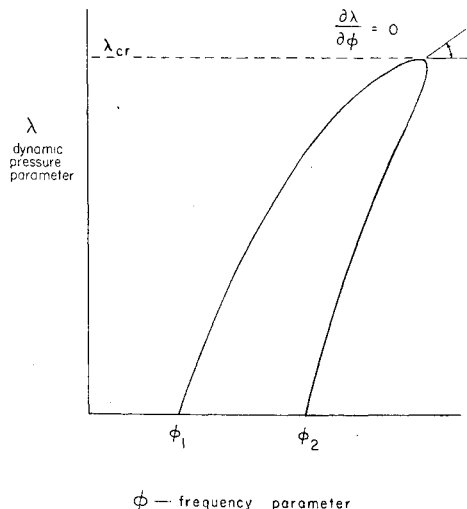


Fig. 2 Schematic diagram of frequency loop—dynamic pressure parameter vs frequency.

axis), the boundary conditions become

$$w = 0, M_x = 0, Q_y = 0 \quad (8)$$

For the streamwise edges, (parallel to x axis) the boundary conditions become

$$w = 0, M_y = 0, Q_x = 0 \quad (9)$$

### Solution of Differential Equations

General product solutions for the lateral deflection and two shear forces which satisfy the boundary conditions on the streamwise edges (Eqs. 9) are chosen as

$$\begin{aligned} w &= A e^{m(x/a)} \sin(\pi/b)y e^{i\omega t} \\ Q_x &= B e^{m(x/a)} \sin(\pi/b)y e^{i\omega t} \\ Q_y &= C e^{m(x/a)} \cos(\pi/b)y e^{i\omega t} \end{aligned} \quad (10)$$

In the previous equations,  $\omega$  is the circular frequency of the panel. The argument of the trigonometric function could have been taken more generally as  $(n\pi/b)y$ , but this would involve carrying an unnecessary  $n$  throughout the rest of the solution. Since the primary flutter speed is of interest,  $n$  is set equal to one. It can easily be replaced at the end of the analysis if more generality is desired. Substituting the assumed solutions (Eqs. 10) into the equilibrium equations and using the usual mathematical argument for a nontrivial solution leads to

$$\begin{vmatrix} -k_x \frac{m^2}{\pi^2} + \theta^2 \phi - \frac{\lambda m \theta}{\pi^4} & \frac{m}{\pi} & -1 \\ \frac{m}{\pi} \theta^2 - \frac{m^3}{\pi^3} - \frac{m}{\pi} \theta^2 J \Omega^2 & -\theta^2 - \frac{1-\mu}{2} r \theta^2 + r \frac{m^2}{\pi^2} & -\frac{1+\mu}{2} \frac{m}{\pi} r \\ \theta^2 - \frac{m^2}{\pi^2} - \theta^2 J \Omega^2 & \left( \frac{1+\mu}{2} \right) \frac{m}{\pi} r & -1 + \frac{1-\mu}{2} \frac{r}{\theta^2} \frac{m^2}{\pi^2} - r \end{vmatrix} = 0 \quad (11)$$

where

$$\begin{aligned} k_x &= (N_x b^2 / \pi^2 D) \quad \theta = (a/b) \quad \phi = \Omega^2 + \bar{N}_y \\ J &= (J b^2 / \pi^2 D) \omega_r^2 \quad \Omega^2 = (\omega / \omega_r)^2 \quad \bar{N}_y = (N_y b^2 / \pi^2 D) \\ \lambda &= (2 q b^3 / \beta D) \quad r = (\pi^2 D / b^2 D_0) \quad \omega_r^2 = (\pi^4 D / b^4 \rho_m) \end{aligned} \quad (12)$$

Evaluating this determinant results in a sixth-degree equation in  $m$  from which six values of  $m$  are found; this is consistent with the original equilibrium equations which are of sixth order in  $w, Q_x, Q_y$ . The sixth-degree characteristic equation is grouped into a quartic part and a quadratic in the following manner where the quadratic part is independent of the frequency parameter

$$(m^4 - 4\gamma m^3 + \pi^2 \bar{A} m^2 + \bar{\lambda} m - \pi^4 \bar{B}) \times \left[ 1 - \frac{r}{\theta^2} \left( \frac{1-\mu}{2} \right) \left( \frac{m^2}{\pi^2} - \theta^2 \right) \right] = 0 \quad (13)$$

where

$$\begin{aligned} \gamma &= [\lambda \theta r / 4 \pi^2 (1 - r k_x)] \\ \bar{A} &= [\theta^2 / (1 - r k_x)] [k_x - 2 + J \Omega^2 + r(\Omega^2 + \bar{N}_y + k_x)] \\ \bar{\lambda} &= \lambda \theta^3 (1 + r) / (1 - r k_x) \\ \bar{B} &= [\theta^4 / (1 - r k_x)] [r(\Omega^2 + \bar{N}_y) + \Omega^2 + \bar{N}_y - 1 + J \Omega^2] \end{aligned} \quad (14)$$

Instead of solving the characteristic equation for the values of  $m$  directly, an indirect approach is taken. The four roots of the quartic part of Eq. (13) are chosen as follows by using Descartes' rule

$$\begin{aligned} m_1 &= \gamma + \alpha + i\delta, m_2 = \gamma + \alpha - i\delta \\ m_3 &= \gamma - \alpha + \epsilon, m_4 = \gamma - \alpha - \epsilon \end{aligned} \quad (15)$$

Matching coefficients of decreasing powers of  $m$  leads to

$$6\gamma^2 - 2\alpha^2 + \delta^2 - \epsilon^2 = \pi^2 \bar{A} \quad (16)$$

$$2\alpha(\delta^2 + \epsilon^2) - 2\gamma[\delta^2 - \epsilon^2 + 2(\gamma^2 - \alpha^2)] = \bar{\lambda} \quad (17)$$

$$\begin{aligned} \gamma^4 + \gamma^2(\delta^2 - \epsilon^2 - 2\alpha^2) - 2\alpha\gamma(\delta^2 + \epsilon^2) + \\ (\alpha^2 - \epsilon^2)(\alpha^2 + \delta^2) = -\pi^4 \bar{B} \end{aligned} \quad (18)$$

For computational purposes, it becomes convenient to solve Eqs. (16) and (17) for  $\delta$  and  $\epsilon$ .

$$\delta^2 = \bar{\lambda} / 4\alpha + \alpha^2 + \pi^2 \bar{A} / 2 - \gamma / \alpha (2\gamma^2 + 3\gamma\alpha - \pi^2 \bar{A} / 2) \quad (19)$$

$$\epsilon^2 = \bar{\lambda} / 4\alpha - \alpha^2 - \pi^2 \bar{A} / 2 - \gamma / \alpha (2\gamma^2 - 3\gamma\alpha - \pi^2 \bar{A} / 2) \quad (20)$$

Using the aforementioned relationships to eliminate  $\delta$  and  $\epsilon$  from Eq. (18) yields the following cubic in  $\alpha^2$ ,

$$\alpha^6 - D_1 \alpha^4 + D_2 \alpha^2 - D_3 = 0 \quad (21)$$

where

$$\begin{aligned} D_1 &= 3\gamma^2 - \pi^2 \bar{A} / 2 \\ D_2 &= (2\gamma^2 - \pi^2 \bar{A} / 2)^2 + \frac{1}{4} \pi^4 \bar{B} - \gamma \bar{\lambda} - \gamma^4 \\ D_3 &= [\bar{\lambda} / 8 - \gamma(\gamma^2 - \pi^2 \bar{A} / 4)]^2 \end{aligned} \quad (22)$$

Equations (19-21) provide a means of obtaining numerical values of the dynamic pressure parameter for corresponding values of frequency once the boundary conditions on the leading and trailing edges are satisfied.

### Boundary Conditions Along Leading and Trailing Edges

Before the boundary conditions can be applied to solutions given by Eqs. (10),  $w, Q_x$ , and  $Q_y$  are written in terms of the

B coefficient. Substituting these expressions for  $w, Q_x$ , and  $Q_y$  into the boundary conditions of Eq. (8), results in six equations in six unknowns. For a nontrivial solution to exist, the determinant of the coefficients is set equal to zero and the values for the roots chosen in Eqs. (15) are substituted into the resulting algebraic equation, yielding

$$\begin{aligned} F(\alpha, \delta, \epsilon, \gamma) &= (\delta^2 + \epsilon^2) + 4\alpha^2(\delta^2 - \epsilon^2) + \\ &4\gamma^2(4\alpha^2 + \delta^2 - \epsilon^2) \sin \delta \sin \epsilon - 8\epsilon \delta (\alpha^2 - \gamma^2) \times \\ &(\cosh \epsilon \cos \delta - \cosh \alpha) = 0 \end{aligned} \quad (23)$$

Now the equilibrium equations and all boundary conditions are satisfied.

### Solution

If for a given plate geometry and elastic stress condition, the necessary previous Eqs. (14, 19-21, and 23) are satisfied then a flutter solution is obtained. For certain values of dynamic pressure parameter ( $\lambda$ ) and frequency parameter ( $\Omega^2$ ), the solutions represent a stable motion of the panel. For  $\lambda = 0$ , the natural frequencies of the plate are obtained for each geometry and loading condition. As the dynamic pressure parameter ( $\lambda$ ) is gradually increased, the value of the vibrational frequency changes. This results in a smooth curve of frequency vs dynamic pressure parameter. As the value of  $\lambda$  is increased, the curves for the lowest two frequencies coalesce at a critical value  $\lambda_{cr}$ . At  $\lambda_{cr}$ , the frequency becomes the complex conjugate of the stable frequency, and at least one of the roots causes the motion to be divergent with time. Below the critical value  $\lambda_{cr}$ , the roots of the frequency equation remain real and the motion is stable provided the buckling load of the panel has not been reached (Fig. 2).

Since the frequency  $\Omega^2$  and the axial load perpendicular to the flow  $N_y$  appear together only in the parameter  $\phi$ , the critical dynamic pressure parameter  $\lambda_{cr}$  is not affected by

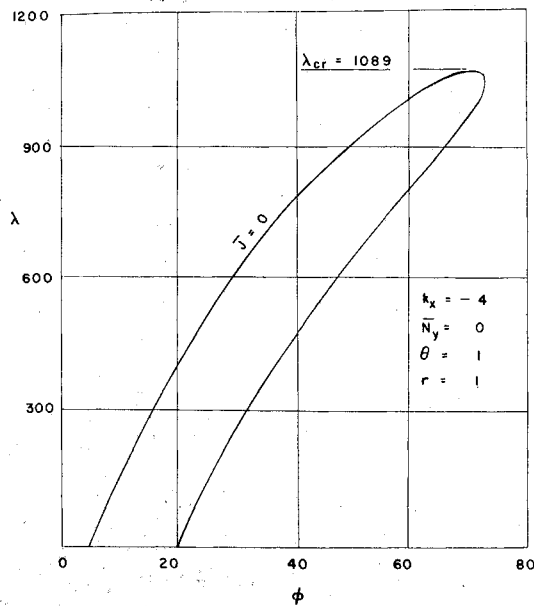


Fig. 3 Frequency loop—dynamic pressure parameter vs frequency for rotary inertia parameter  $J = 0$ .

the value of  $N_y$ . The value of  $N_y$  does change the values of the natural frequencies and thus the starting points for the flutter loops. Different values of  $N_y$  result in a shift of the flutter loops along the frequency axis, but do not change the critical dynamic pressure parameter  $\lambda_{cr}$ . This was also found to be true in the works of Hedgepeth<sup>2</sup> and Erickson and Anderson.<sup>6</sup> As  $N_y$  is increased from zero, the value of the natural frequency decreases.

The natural frequency of the system is obtained by setting the dynamic pressure parameter  $\lambda$  equal to zero which results in  $\gamma$  and  $\bar{\lambda}$  equal to zero (Eq. 14)

$$\Omega^2 = \frac{\left(\frac{m^2}{\theta^2} + 1\right)^2}{1 + r \left(\frac{m^2}{\theta^2} + 1\right) + J \left(\frac{m^2}{\theta^2} + 1\right)} - \frac{\left(\frac{m}{\theta}\right)^2 \frac{\left(\frac{m^2}{\theta^2} r + r + 1\right) k_x}{1 + r \left(\frac{m^2}{\theta^2} + 1\right) + J \left(\frac{m^2}{\theta^2} + 1\right)} - \frac{\left(1 + r + r \frac{m^2}{\theta^2}\right) N_y}{1 + r \left(\frac{m^2}{\theta^2} + 1\right) + J \left(\frac{m^2}{\theta^2} + 1\right)} \quad (24)$$

It becomes apparent in Eq. (24) that if  $r = 0$ , that is, if the plate is homogeneous and isotropic, and the rotary inertia parameter  $J$  is taken equal to zero as was done by Hedgepeth, the equation reduces to the frequency equation for an ordinary plate with in-plane loads in the  $x$  and  $y$  direction, where  $m$  represents the  $m^{\text{th}}$  mode in the flow direction. Note also that if the frequency parameter  $\Omega^2$  is set equal to zero, the buckling loads for an in-plane loaded plate can be obtained.

## Results

In this analysis, the value of  $\lambda_{cr}$  is determined by plotting values of  $\lambda$  vs frequency  $\phi$  and obtaining the maximum point of this curve. Values of  $r, k_x, \theta, \bar{N}_y, J$  are varied to obtain a spectrum of results for different geometric configurations

and stress conditions. The results may be reduced to those presented by Erickson and Anderson<sup>6</sup> by setting the rotary inertia parameter,  $J$ , equal to zero (Fig. 3). Also, it is possible to obtain results of Hedgepeth<sup>2</sup> by setting both  $J$  and  $r$  equal to zero. If  $r = 0$ , the plate has infinite shear stiffness or is an ordinary solid plate.

The graph of Fig. 4 shows that the critical dynamic pressure parameter decreases as the rotary inertia parameter increases. The plate parameters  $\theta = 1, k_x = -4, r = 1$ , and  $\bar{N}_y = 0$  are plotted for varying values of the rotary inertia parameter  $J$ . An interesting observation is that the natural frequencies shift to the left as the value of  $J$  increases. As the rotary inertia term becomes large, the frequencies decrease to values lower than the natural frequencies obtained if rotary inertia was neglected. These lower natural frequencies are starting points for the flutter curves which cause the flutter loop to reach a maximum value below those values where rotary inertia is neglected. Naturally as the value of  $J$  approaches zero, the natural frequencies for rotary inertia and those neglecting rotary inertia terms coincide and the results obtained by Erickson and Anderson<sup>6</sup> are duplicated.

Erickson and Anderson<sup>6</sup> obtained curves for the critical dynamic pressure parameter for a given plate and stress condition while varying the value of the shear flexibility parameter  $r$ . It was observed that for high-axially tensioned plates, as the shear stiffness  $D_Q$  decreased, the critical dynamic pressure parameter increased. This result can be observed in Figs. 5 and 6 for  $J = 0$ . This result says essentially that it takes a higher critical dynamic pressure parameter to cause flutter for decreasing shear stiffness.

Some results obtained from this analysis can be seen on Figs. 5 and 6 for representative values of the rotary inertia parameter  $J$ . It can be observed that for given values of  $r$  and increasing values of  $J$ , the critical dynamic pressure parameter is decreased for each plate configuration. However, the result of requiring a higher dynamic pressure parameter as the shear stiffness  $D_Q$  decreases is still obtained by using this analysis. For each value of  $J$ , the curves rise as the value of  $r$  is increased except for an initial dip which occurs at low value of  $r$ . In Fig. 5, however, the curves plotted for increasing values of  $J$  seem to become asymptotic to lines parallel to the  $r$  axis as the value of  $J$  increases.

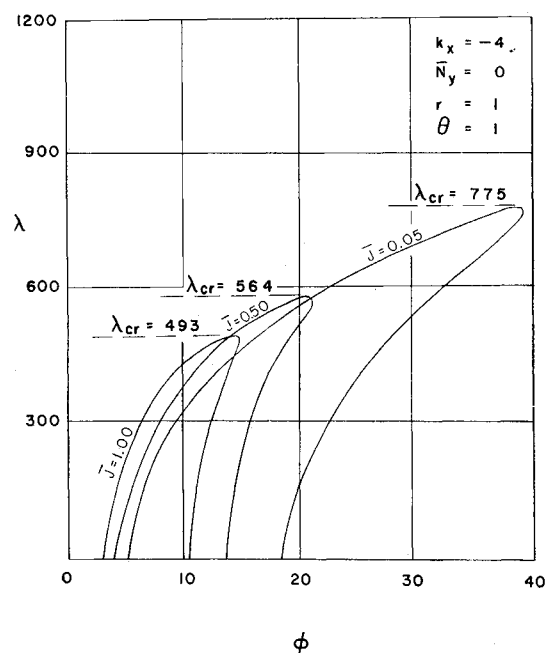


Fig. 4 Effect on dynamic pressure parameter for varying values of rotary inertia parameter.

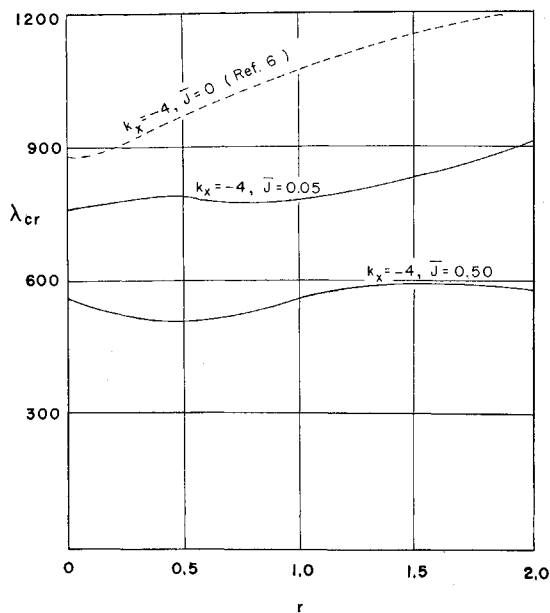


Fig. 5 Critical dynamic pressure parameter vs shear flexibility parameter for varying values of rotary inertia parameter; axial load parameter  $k_x = -4$ .

### Conclusions

The analysis and results lead to the following conclusions: 1) for some values of the rotary inertia parameter, the critical dynamic pressure parameter  $\lambda_{cr}$  increases as the shear stiffness  $D_Q$  decreases; 2) increasing the rotary inertia parameter causes the critical dynamic pressure parameter  $\lambda_{cr}$  to decrease for a given value of the shear flexibility parameter; 3) increasing the rotary inertia parameter causes the natural frequencies to decrease. When the rotary inertia terms are dropped from the equations, the natural frequencies are identical to those obtained by previous investigators; 4)  $N_y$  in-plane loading (in the cross-stream direction) shifts the flutter loop along the frequency axis and does not change the critical value of the dynamic pressure parameter. The principal conclusion of this investigation concerns the anomalous behavior of increasing critical dynamic pressure parameter for decreasing shear stiffness  $D_Q$ . Results obtained by Erickson and Anderson are now verified by a more inclusive analysis. It is believed from theoretical considerations which adequately describe the physical system that this behavior occurs. Experimental investigation of flutter at high-in-plane tensile loads acting in the stream direction would provide another means of correlation. Therefore, it is recommended that future efforts be directed toward experimental proof of these theoretical results. The results presented here should provide a good

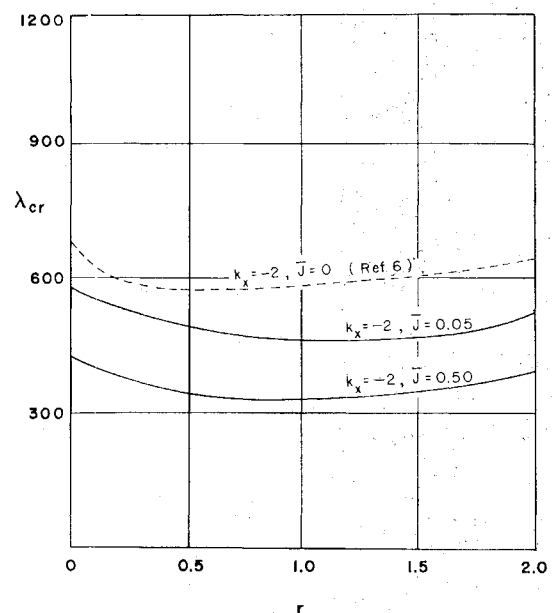


Fig. 6 Critical dynamic pressure parameter vs shear flexibility parameter for varying values of rotary inertia parameter; axial load parameter  $k_x = -2$ .

basis for evaluation of an experimental flutter program for simply-supported sandwich plates including rotary inertia effects.

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